

Deviation From Λ CDM With Cosmic Strings Networks

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Abstract

In this work, we consider frustrated network of cosmic strings to explain possible deviation from Λ CDM behaviour. We use different observational data to put constraint on the model and show that a small but non zero contribution from the string network is allowed which can explain the possible small departure from Λ CDM evolution. By calculating the Bayesian Evidence, we show that our model and the concordance Λ CDM model are equally favored by the observational data.

Keywords: Dark Energy, Cosmic Strings, SnIa, CMB, BAO.

1. Introduction

One of the greatest mysteries in cosmology today is the nature of dark energy in the Universe. According to the concordance cosmology, this makes up around 70% of the total energy density of the universe and causes the universe to accelerate around the present epoch [1]. Yet, there is no strong theoretical as well as observational indication to pin point the nature of this dark component. In fact we still do not know whether the mysterious late time acceleration is due to the presence of dark energy or due to any modification of gravitational laws on large scales.

Nevertheless, majority of the observational data [2, 3, 5, 4] support the concordance Λ CDM model as a possible explanation for the late time acceleration. In this model, the presence of the cosmological constant (Λ) poses two serious problems. First one is related to the fine tuning of the value of Λ to a ridiculously small number ($\sim 10^{-120} M_{pl}^4$ in natural units) necessary for the present day cosmic acceleration. The other one is related to its dominance over the matter component of the universe precisely at the present epoch. Any small departure from these two conditions will result a cosmological evolution that is different from the observable Universe.

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These conceptual problems dilute the superiority, the Λ CDM model enjoys over other dark energy models as far as observational results are concerned.

Interestingly, current observational data also allow deviation from the Λ CDM behaviour. Although the deviation is not large, but it is still detectable with the current and future precisions of the observational set ups. Hence efforts are on to construct models that can explain this deviation. Almost all the approaches for this purpose, assume that Λ is exactly zero in our universe, and an evolving dark energy is solely responsible for the late time acceleration of the universe. Possibilities for this include quintessence [6], k-essence [7], an arbitrary barotropic fluid like Chaplygin gas or its various generalization and many more [8]. All these models are generally phenomenological without any strong theoretical motivations and they also burden us with the challenge of explaining several other model parameters.

The other option is to assume the existence of a small but non-zero Λ as in Λ CDM model but to consider some extra component to explain the observed deviation from Λ CDM behaviour [9]. This is motivated by the fact that there is no mechanism available that can make Λ vanish without any conflict with the meaningful physics [10]. Moreover the string-landscape model may explain the existence of a small but non zero Λ in our present universe [11]. But it is important to have well motivated candidate that can explain the deviation from Λ CDM; otherwise this approach will have the same shortcomings that are present in standard dark energy models.

In this study, we propose for such a model. There are models where dark energy is assumed to be a perfect fluid having equation of state $w = \frac{p}{\rho} < 0$. But one major problem for such models is due to its sound velocity $c_s^2 = w < 0$ which causes instabilities on the small scales. It was then proposed to include rigidity in the fluid to make it an elastic solid [12]. In this case, if the rigidity is sufficiently large, $c_s^2 > 0$ even if $w < 0$.

A solid perfect fluid can arise in different theories. One such possibility is the frustrated network of cosmic strings [13, 15, 16]. These are topological defects formed during the phase transitions in the early universe and can survive till present day. If these strings are produced during the phase transition at the electroweak scale, they are very light. But there can be many such strings and their combined energy density can be finite. If the cosmic strings intercommute, their evolution obeys a scaling solution. In that case, the energy density scales as a^{-3} in the matter dominated era or a^{-4} in the radiation dominated era. In such scenario, strings can never accelerate the universe. On contrary, if the strings do not intercommute or pass through each other, then the network can be frozen in comoving coordinates. These are called frustrated network of strings. Such network of strings will have the equation of state $w = -1/3$ and hence their energy density will go as $\rho_s \sim a^{-2}$. A phase transition around few TeV and a string separation today of the order of few A.U. will result Ω_s today in the range of 0 and 1 [13]. So it is possible to get a universe containing frustrated network of cosmic strings that can explain the possible small deviation from the Λ CDM. For this we do not need Ω_s today to dominate

the energy budget of the universe; a small but non zero value can be sufficient for this purpose. This is certainly achievable from a well motivated particle physics set up. Also note that these network of strings are not theoretical fantasies; they can be seen in the biaxial nematic liquid crystals [17]. Obviously the question is whether in any reasonable particle physics model, we get a frustrated network of strings. It was shown by several numerical simulations that abelian string networks intercommute and give rise to scaling solution without frustration [18]. But later on it was shown that in non-abelian string network, intercommutation can be prevented due to topological constraints and they can form stable frustrated network [13].

There is also another simple way that can result a string-like term ($\rho_s \sim a^{-2}$) in the Hubble equation. A fluid with a constant negative pressure ($p = -p_0$, with p_0 being a positive constant) behaves like a Λ CDM model. This can be easily seen by putting the constant p in the energy conservation equation $\dot{\rho} + 3H(\rho + p) = 0$ and then integrate it out to obtain the expression for ρ . Let us model small deviation from the Λ CDM behaviour around present day ($z = 0$) by Taylor expanding around this constant p behaviour:

$$p = -p_0 + p_1 z + p_2 z^2, \quad (1)$$

where we keep terms upto second order. p_1 and p_2 are related to $\frac{dp}{dz}|_{z=0}$ and $\frac{d^2 p}{dz^2}|_{z=0}$ respectively. With this one can integrate the energy conservation equation to get the expression for energy density and pressure as a function of scale factor:

$$\begin{aligned} \rho(a) &= (p_0 + p_1 - p_2) + \frac{c}{a^3} - \frac{3}{2} \frac{p_1 - 2p_2}{a} - \frac{3p_2}{a^2} \\ p(a) &= -(p_0 + p_1 - p_2) + \frac{p_1 - 2p_2}{a} + \frac{p_2}{a^2}. \end{aligned} \quad (2)$$

Here c is an arbitrary integration constant. For the choice $p_1 = 2p_2$ and $p_2 < 0$, these gives a mixture of fluids containing Λ , matter and a frustrated network of cosmic strings.

Hereafter we consider such a mixture of fluids and discuss its various cosmological implications. We start with the Einstein equations:

$$\begin{aligned} \frac{H^2}{H_0^2} &= \Omega_{m0} a^{-3} + \Omega_\Lambda + \Omega_{s0} a^{-3(1+w_s)} \\ 2\dot{H} + 3H^2 &= -3H_0^2 [w_s \Omega_{s0} a^{-3(1+w_s)} - \Omega_\Lambda] \end{aligned} \quad (3)$$

Here subscript "0" means values at present ($z = 0$, $a_0 = 1$), w_s is the equation of state for the network of strings. Flatness condition demands that $\Omega_{m0} + \Omega_\Lambda + \Omega_{s0} = 1$; hence two of these parameters are independent. Henceforth we shall consider Ω_{m0} and Ω_{s0} . As we have discussed in the previous paragraphs, for a frustrated network of cosmic strings $w_s = -1/3$. Usually the

equation of state of a cosmic string network is related to its root-mean squared (rms) velocity as [14]

$$w_s = \frac{1}{3}(2v_s^2 - 1). \quad (4)$$

In the limit $v_s \rightarrow 0$, one gets the equation of state for string network as $w_s = -1/3$. It has been shown through different numerical simulations that in the string dominated universe $v_s^2 \leq 0.17$ [19] but can never be exactly zero for nonintercommutating strings. In this case, it can be shown that the string network can never be conformally stretched to the Hubble scale preventing them to fraustrate. But in our set up, the energy density of the string network is subdominant and never governs the expansion of the universe. Hence even if $v_s^2 \neq 0$, the correlation length can grow much slower than the expansion rate of the universe and the network can be conformally stretched to Hubble scale and can fraustrate. This possibility was also discussed by Sousa and Avelino [20]. In our subsequent study, we assume the parameter v_s^2 to be non zero, and see how far one can constrain this parameter together with the other model parameters like w_s , Ω_{m0} and Ω_{s0} using the presently available observational data.

The equation of state for the combined dark energy fluid (Λ +Cosmic String) is now given by

$$w_{de} = \frac{\Omega_{m0} + \Omega_{s0} - 1 + \frac{1}{3}(2v_s^2 - 1)\Omega_{s0}a^{-2(1+v_s^2)}}{1 - \Omega_{m0} - \Omega_{s0} + \Omega_{s0}a^{-2(1+v_s^2)}}. \quad (5)$$

Later, we reconstruct this dark energy equation of state, w_{de} , to see how much deviation from $w = -1$ is allowed by the current cosmological observations.

As we discussed in previous paragraphs, if we consider this network of strings as an elastic fluid by introducing rigidity, one can get $c_s^2 > 0$ for this elastic fluid which will ensure stability in the small scale perturbations. It has been shown by Battye et al. [21] that under reasonable but not totally general circumstances, one can obtain $c_s^2 = 1/5$ for a elastic network of cosmic strings. In our subsequent calculations, we shall assume this value for c_s^2 for the cosmic strings.

With this dark energy set up, we use the present observational data to constrain our model. We begin by considering the Type Ia Supernovae observation which probes directly the cosmological expansion. These observations measure the apparent luminosity of the Supernova explosion as observed by an observer on earth. In cosmological terms, this is given by luminosity distance $d_L(z)$ defined as

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')} \quad (6)$$

Using this, we evaluate the distance modulus μ (which is an observable quantity) as

$$\mu = m - M = 5 \log \frac{d_L}{Mpc} + 25, \quad (7)$$

where m and M are the apparent and absolute magnitudes of the Supernovae respectively. We consider the latest Union2.1 data compilation [3] consisting of 580 data points for the observable μ .

Next, we use the observational constraints on Hubble parameter as available in the literature. Recently Moresco et al. [22] have compiled the Hubble parameter measurements in the redshift range $0 < z < 1.75$ using the differential evolution of the cosmic chronometers. They have built a sample of 18 observational data point for $H(z)$ spanning almost 10 Gyr of cosmic evolution. These values are given in Table 1. To complete the data set, we also use the latest and the most precise measurement of the Hubble constant H_0 [5].

z	$H(z)$	$\sigma_{H(z)}$	Ref.
0.090	69	12	[23]
0.170	83	8	[23]
0.179	75	4	[22]
0.199	75	5	[22]
0.270	77	14	[23]
0.352	83	14	[22]
0.400	95	17	[23]
0.480	97	62	[24]
0.593	104	13	[22]
0.680	92	8	[22]
0.781	105	12	[22]
0.875	125	17	[22]
0.880	90	40	[24]
1.037	154	20	[22]
1.300	168	17	[23]
1.430	177	18	[23]
1.530	140	14	[23]
1.750	202	40	[23]

Table 1: $H(z)$ measurements (in units $[\text{km s}^{-1}\text{Mpc}^{-1}]$) and their errors.

Next, we use the combined BAO/CMB constraints as derived recently by Giostri et al. [28]. For this, one starts defining the comoving sound horizon at the decoupling as:

$$r_s(z_*) = \frac{1}{\sqrt{3}} \int_0^{1/(1+z_*)} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_{b0}/4\Omega_{\gamma0})a}}, \quad (8)$$

where $\Omega_{\gamma0}$ and Ω_{b0} are the photon and baryon density parameter respectively at present ($z = 0$) and we have assumed $c = 1$. Here z_* is the redshift at decoupling and is accurately given by the formula as obtained in [29]. In accordance with WMAP7 [26], we put $z_* = 1091$ exactly. Another important quantity is the redshift of the drag epoch ($z_d \approx 1020$), when the photon pressure is no longer able to avoid gravitational instability of the baryons.

Next we define the "acoustic scale":

$$l_A = \pi \frac{d_A(z_*)}{r_s(z_*)}, \quad (9)$$

where $d_A(z_*) = \int_0^{z_*} dz'/H(z')$ is the comoving angular-diameter distance. One can also define the "dilation scale" as introduced in [4]:

$$D_V(z) := [d_A^2(z)z/H(z)]^{1/3}. \quad (10)$$

The 6dF Galaxy Survey and more recently, the WiggleZ team [27] measured this quantity at 0.106, $z = 0.44$, $z = 0.60$ and $z = 0.73$. Percival et al. [30] also measured $\frac{r_s(z_d)}{D_V(z)}$ at $z = 0.2$ and $z = 0.35$. Combining these measurements with the WMAP-7 measurement of l_a [26], one can obtain the combined measurement of BAO/CMB for the quantity $\left(\frac{d_A(z_*)}{D_V(z_{BAO})}\right)$. This has been given in [28]. From this one can obtain

$$\chi_{BAO/CMB}^2 = \mathbf{X}^t \mathbf{C}^{-1} \mathbf{X}, \quad (11)$$

where

$$\mathbf{X} = \begin{pmatrix} \frac{d_A(z_*)}{D_V(0.106)} - 30.95 \\ \frac{d_A(z_*)}{D_V(0.2)} - 17.55 \\ \frac{d_A(z_*)}{D_V(0.35)} - 10.11 \\ \frac{d_A(z_*)}{D_V(0.44)} - 8.44 \\ \frac{d_A(z_*)}{D_V(0.6)} - 6.69 \\ \frac{d_A(z_*)}{D_V(0.73)} - 5.45 \end{pmatrix} \quad (12)$$

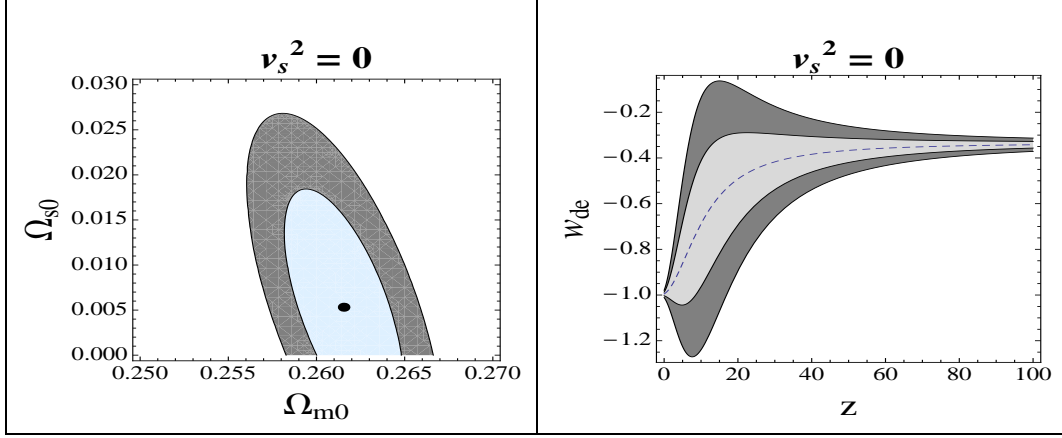


Figure 1: Left Figure: The 1σ and 2σ contour regions in the $\Omega_{m0} - \Omega_{s0}$ plane. The "dot" represents the best fit value. Right Figure: The reconstructed w_{de} behaviour as a function of redshift. The dashed line is the best fit behaviour. The light grey region is for 1σ confidence level and the dark grey region is for 2σ confidence level

and

$$C^{-1} = \begin{pmatrix} 0.48435 & -0.101383 & -0.164945 & -0.0305703 & -0.097874 & -0.106738 \\ -0.101383 & 3.2882 & -2.45497 & -0.0787898 & -0.252254 & -0.2751 \\ -0.164945 & -2.45497 & 9.55916 & -0.128187 & -0.410404 & -0.447574 \\ -0.0305703 & -0.0787898 & -0.128187 & 2.78728 & -2.75632 & 1.16437 \\ -0.097874 & -0.252254 & -0.410404 & -2.75632 & 14.9245 & -7.32441 \\ -0.106738 & -0.2751 & -0.447574 & 1.16437 & -7.32441 & 14.5022 \end{pmatrix} \quad (13)$$

is the inverse covariance matrix derived by using the above results together with the correlation coefficients $r = 0.337$, $r = 0.369$ and $r = 0.438$ calculated for the r_s/D_V pair of measurements at $z = (0.2, 0.35)$, $z = (0.44, 0.6)$ and $z = (0.6, 0.73)$, respectively [30, 27].

We further use the measurement of CMB anisotropy by WMAP-7 observations [5]. Using the publicly available code CAMB [32], we calculate the expected CMB anisotropy spectrum for our model. To incorporate the dark energy perturbation, we use publicly available PPF Module [33] for CAMB. With this we fit our three model parameters Ω_{m0} , Ω_{s0} and v_s^2 . The other cosmological parameters are equated to their best fit values as derived from the WMAP-7 data. We compare our theoretical results with the data and do the likelihood analysis using the likelihood code provided by the WMAP team [5].

With these four set of observational data, we constrain our model parameters and also reconstruct the dark energy equation of state w_{de} . The results are shown in Figures (1), (2) and (3) where we have set v_s^2 equal to 0, 0.1 and 0.17 respectively which corresponds to $w_s = -1/3, -0.27$

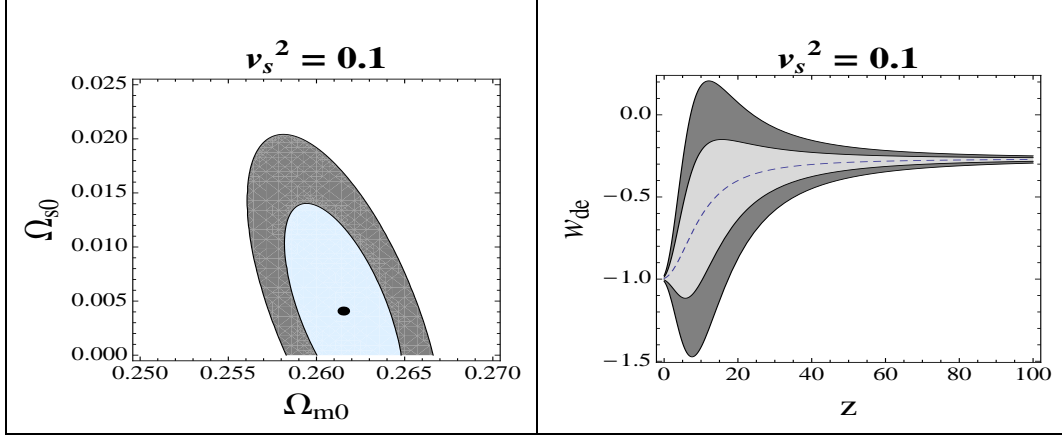


Figure 2: Left Figure: The 1σ and 2σ contour regions in the $\Omega_{m0} - \Omega_{s0}$ plane. The "dot" represents the best fit value. Right Figure: The reconstructed w_{de} behaviour as a function of redshift. The dashed line is the best fit behaviour. The light grey region is for 1σ confidence level and the dark grey region is for 2σ confidence level

and -0.22 respectively. With increasing value of v_s^2 , the allowed range for Ω_{s0} decreases, whereas the constraint on Ω_{m0} does not change appreciably. Also the contribution from the string network is always sufficiently small than the rest of the energy density, ensuring that the string network never dominates the energy density of the universe. The small contribution from Ω_{s0} results the w_{de} to be very close to $w = -1$ at present which is evident in the reconstructed behaviour for w_{de} . As we go past, the equation of state for the dark energy can change appreciably from the $w = -1$ behaviour. In fact for higher values of v_s^2 , it is possible that w_{de} can also scale the matter component ($w_{de} = 0$) at 2σ confidence level.

To summarise, the current observational data is fully consistent with the presence of a small but non zero contribution from cosmic string networks. This small contribution is enough to produce appreciable change in the behaviour of dark energy equation of state w_{de} at earlier times ($z > 0$) although one can not distinguish it from $w = -1$ at the present day.

Next we calculate the Bayesian Evidence for our model. This is defined as [34]

$$E = \int \mathcal{L}(\theta) P(\theta) d\theta, \quad (14)$$

where θ represents the set of model parameters, \mathcal{L} represents the Likelihood function and $P(\theta)$ is the prior probability distribution for parameter θ . According to Jeffrey's interpretation [35], if $\Delta \ln E$ between 1 and 2.5, it is a significant evidence for a model with higher E while the same between 2.5 and 5 is a strong to very strong evidence. If $\Delta \ln E$ is more than 5, the model with higher E is decisively favoured.

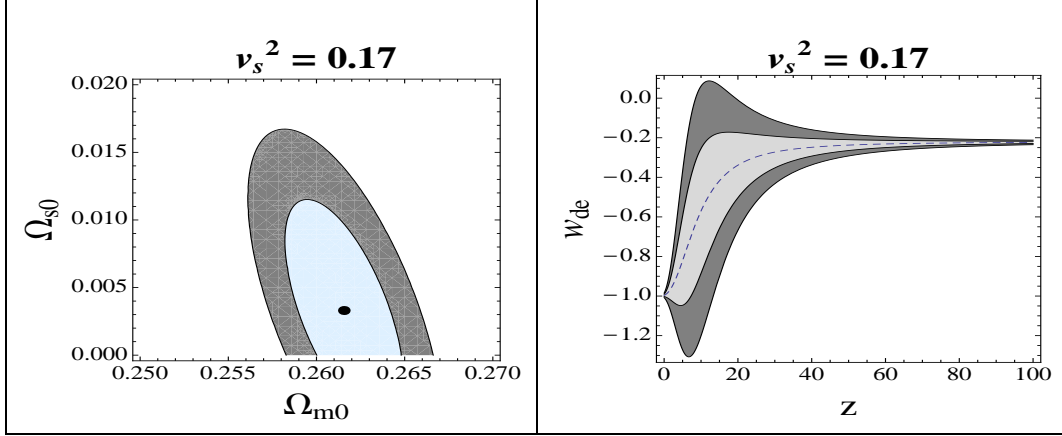


Figure 3: Left Figure: The 1σ and 2σ contour regions in the $\Omega_{m0} - \Omega_{s0}$ plane. The "dot" represents the best fit value. Right Figure: The reconstructed w_{de} behaviour as a function of redshift. The dashed line is the best fit behaviour. The light grey region is for 1σ confidence level and the dark grey region is for 2σ confidence level.

We calculate the evidence for the simple cosmic string network model for which $v_s^2 = 0$ and $w_s = -1/3$. With such choice, we have two model parameters, Ω_{m0} and Ω_{s0} . We assume uniform prior to these parameters. With this the $\Delta \ln E$ for our model and a simple Λ CDM model is 0.97 which shows that evidence-wise our model is equally viable as the concordance Λ CDM model.

To conclude, we propose a model that can explain the deviation from the concordance Λ CDM behaviour. The majority of the approaches for such a model assume the Λ term to be zero and introduce dynamical fields that can give rise to late time acceleration of the universe. All these models are phenomenological in nature and lacks theoretical understanding (See [36] for recent attempt to build a quintessence model in string theory). They not only inherit the problems present in the concordance Λ CDM model, but also burden us with a number of extra parameters which also have to be fine tuned. This motivates us to look for alternative approaches. We propose that the presence of frustrated network of cosmic strings together with the already existing Λ can mimick a suitable dark energy model which is consistent with the observational data. Although this model shares all the difficulties that are present in a Λ CDM model, but the inclusion of cosmic string network which can cause the observed deviation from the Λ CDM behaviour can be justified as we expect such defect networks to be formed during the phase transition in the early universe.

We put constraint on the model parameters using the observational data and the reconstructed equation of state for the dark energy, w_{de} , shows that, although at present, it is indistinguishable from $w = -1$, as one goes to the past, it deviates sufficiently from this behaviour. The interesting fact is that to get such large deviation, one needs a very small contribution from

the cosmic string network, preventing it to dominate the energy budget of the universe.

Finally by using Bayesian Evidence, we show that statistically our model and Λ CDM model are equally favored.

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